A Near-Far Resistant Algorithm to Combat Effects of Fast Fading in Multi-User DS-CDMA Systems

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Abstract

A sliding window decorrelating algorithm is developed. It is shown to be near-far resistant both in AWGN and fast fading environments. The algorithm is of particular relevance to base stations in Multi-User DS-CDMA mobile radio networks, and is shown to alleviate the requirement for a complex power control algorithm to compensate for rapidly varying relative user energies.

1 Introduction

Along with the recent surge of interest in the application of DS-CDMA to mobile radio networks, the near-far problem has come to be accepted as the principal shortcoming of DS-CDMA and the conventional linear correlation receiver. Contrary to popular interpretation of the problem, the near-far effect is due not only to variations in the geographical positioning (and hence propagation loss) of the transmitters, but also to fast fading with variations of the order of tens of decibels in tens of milliseconds. It has been shown [1] that DS-CDMA without power control is rendered unusable as a multiple access (MA) technique due to fast fading. Furthermore many researchers [1, 2] have stated that it is unlikely that power control could compensate for fast fading effects at vehicle speeds found in practice.

Two solution concepts have emerged from the literature. One is based on the optimum multi-user detector [3] and approximations to it [4]. Both these detectors require knowledge of the user received energies, and furthermore the optimum detector has computational complexity exponential in the number of users. These factors preclude the application of this approach in practical systems. The second approach is based on the class of linear detectors defined by Lupas and Verdu [5]. For reasons discussed below the resulting detectors though optimal do not lend themselves to practical implementation.

2 System Model and Near-Far Resistance

We recall the multi-user communication model [3], and use the following terminology. Let $b_k(i)$, $w_k(i)$, and $y_k(i)$ be respectively the i^{th} data bit, the received energy of this data bit, and the corresponding matched filter output of the k^{th} user. Also let $s_k(t)$ and τ_k be the signature waveform and random delay of the k^{th} user respectively. We consider a system consisting of K users and a transmission of arbitrary length N. Finally let $n(t)$ be the AWGN with noise power σ^2. We number the users according to their delays so that

$$0 \leq \tau_1 \leq \tau_2 \ldots\ldots \leq \tau_K < T \tag{1}$$

Also define

$$\tilde{s}_k(t) = \begin{cases} s_k(t) & \text{if } 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Let $R(l)$ be the $K * K$ normalized signal cross-correlation matrices such that

$$R_{k,j}(l) = \int_{-\infty}^{\infty} \tilde{s}_k(t-\tau_k)\tilde{s}_j(t-lT-\tau_j)dt \tag{3}$$

It follows that $R(l) = 0$ for $\mid l \mid > 1$ and $R(-l) = R^T(l)$.

The DS modulated received signal after down-conversion and demodulation can be written as

$$r(t) = \sum_{i=1}^{N}\sum_{k=1}^{K} b_k(i)\sqrt{w_k(i)}\tilde{s}_k(t-iT-\tau_k) + n(t) \tag{4}$$

The matched filter output $y_k(i)$ is then :

$$y_k(i) = \int_{iT+\tau_k}^{iT+T+\tau_k} r(t)\tilde{s}_k(t - iT - \tau_k)dt \qquad (5)$$

We use vector notation for the matched filter outputs, transmit data and noise (eg $\underline{y}(l) = [y_1(l), y_2(l), \ldots, y_k(l)]$), and also define a transmit energy matrix $W(l) = diag\left(\sqrt{w_1(l)}, \sqrt{w_2(l)}, \ldots, \sqrt{w_k(l)}\right)$. Substituting we have

$$\underline{y}(l) = R(-1)W(l+1)\underline{b}(l+1) + R(0)W(l)\underline{b}(l) + R(1)W(l-1)\underline{b}(l-1) + \underline{n}(l) \qquad (6)$$

where $\underline{n}(l)$ is the matched filter output noise vector at time instant $t = lT$ with auto-correlation matrix given by :

$$E\left[n(i)n^T(i)\right] = \sigma^2 R(i-j) \qquad (7)$$

It must be noted that a single element of $\underline{y}(l)$ consists of the following components:

1. The required signal $\sqrt{w_k(l)}b_k(l)$.
2. Interference from $l+1^{th}$, l^{th} and $l-1^{th}$ bits of other users.
3. correlated additive noise.

Near-Far Resistance is defined in [5]. In the context of a multi-user receiver it can be stated as follows :

A receiver is **near-far resistant** if *the BER of a particular user goes to zero as the thermal noise PSD is reduced to zero, irrespective of the number of users in the system or their relative energies* [5]. Clearly the scope of this definition is limited to AWGN channels as fading and mobile radio channels in general have irreducible error rates due to Doppler effects.

3 Development of Sliding Window Algorithm

The discrete model of (6) can be written as

$$\begin{bmatrix} \underline{y}(M+1) \\ \underline{y}(M+2) \\ \underline{y}(M+3) \\ \vdots \\ \underline{y}(M+\acute{N}) \end{bmatrix} = \begin{bmatrix} R(0) & R(-1) & 0 & \cdots & 0 \\ R(1) & R(0) & R(-1) & \cdots & 0 \\ 0 & R(1) & R(0) & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & R(1) & R(0) \end{bmatrix} \mathcal{W} \begin{bmatrix} \underline{b}(M+1) \\ \underline{b}(M+2) \\ \underline{b}(M+3) \\ \vdots \\ \underline{b}(M+\acute{N}) \end{bmatrix} + \mathcal{N} \qquad (8)$$

where

$$\mathcal{N}^T = \left[\underline{n}(M+1), \underline{n}(M+2), \underline{n}(M+3), \cdots, \underline{n}(M+\acute{N})\right]$$

and

$$\mathcal{W} = diag\left[W(M+1), W(M+2), W(M+3), \cdots, W(M+\acute{N})\right]$$

where M is an offset from the start of the window, and $\acute{N} < N$ ie we have a window of data within the entire received sequence. Since both the energies and the bit values are unknown at the receiver, we seek both these values. From (6) $\underline{y}(M+1)$ depends on $\underline{b}(M)$ and similarly for $\underline{y}(M+\acute{N})$ and $\underline{b}(M+\acute{N}+1)$. Hence the linear system (7) is invalid as it stands unless $M = 0$ and $\acute{N} = N$. This special case clearly reduces to the finite sequence length case discussed in [5]. Also $\Re$ is $NK * NK$, and its inversion is not feasible for practical sequence length values and makes unacceptable demands on the level of user cooperation. Lupas and Verdu propose a LTI filter implementation which involves the inversion of a $K * K$ matrix, each element of which is a polynomial of the form $az^{-1} + b + cz$. This approach is impractical for large K and is also inflexible to changes in the timing configuration and to the addition/removal of users.

From now on we assume an infinite length data sequence. This places no demands on the cooperation between users in the system. Our strategy is to partition the infinite sequence into blocks of size $\acute{N}$. Here $\acute{N}$ is such that the solution of the linear system is feasible within a time period of the order of one data bit period. We first define a valid linear system which can be solved within a finite data window (rather than the entire data sequence). By sliding the processing window over the incoming data (Figure 1), we can process a data transmission of infinite length on a real time basis with a constant delay of the order of a data bit period. Define a continuous valued transmit data vector as follows.

$$\underline{WB}(i) = \left[\sqrt{w_1(i)}b_1(i), \sqrt{w_2(i)}b_2(i), \ldots, \sqrt{w_K(i)}b_K(i)\right]$$

It is clear that the required correction terms are $R(1)\underline{WB}(M)$ in the case of $\underline{y}(M+1)$, and $R(-1)\underline{WB}(M+\acute{N}+1)$ in the case of $\underline{y}(M+\acute{N})$.

We define a modified input vector X by

$$\begin{aligned} \underline{x}(M+1) &= \underline{y}(M+1) - R(1)\underline{WB}(M) \\ \underline{x}(M+\acute{N}) &= \underline{y}(M+\acute{N}) - R(-1)\underline{WB}(M+\acute{N}+1) \\ \underline{x}(M+i) &= \underline{y}(M+i) \text{ for } 1 < i < \acute{N} \end{aligned} \qquad (9)$$

We have reduced our problem to

1. Obtaining reasonable estimates of the terms $\underline{WB}(M)$ and $\underline{WB}(M+\acute{N}+1)$
2. Solving the linear system $\underline{X} = \Re\mathcal{W}\underline{b} + \underline{n}$, $\Re$ block tri-diagonal.

We now consider each item within our estimation problem. $\underline{WB}(M)$ is readily available provided the receiver

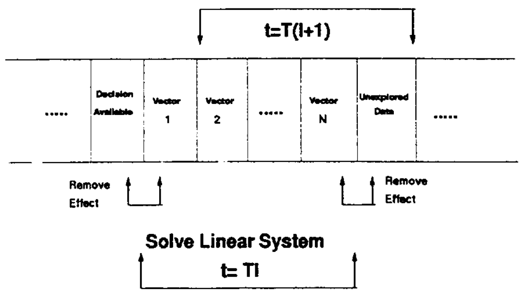

Figure 1: Schematic Representation of Sliding Window Decorrelator

is initialised in a suitable manner, since the decorrelating problem would have already been solved for this data item. Clearly our estimate holds with equality when the noise power spectral density (NPSD) is zero. Hence we conform to the near-far resistant condition when making this estimate. As regards the prediction of the unexplored data, the only information available to aid the estimation of $\underline{WB}(M + \acute{N} + 1)$ is the corresponding vector of matched filter outputs. Since we assume that MA interference is the dominant source of interference, and underpins our investigation, we cannot approximate $\widehat{\underline{WB}}(M+\acute{N}+1)$ by $\underline{y}(M+\acute{N}+1)$. We use the following approximation for the future received energy vector.

$$\underline{W}(M + \acute{N} + 1) \approx \hat{\underline{W}}(M + \acute{N} - 1) \qquad (10)$$

We are hence left with the problem of predicting the polarity of the future data bit $\underline{b}(M + \acute{N} + 1)$. The simplest approach would be to use the matched filter outputs corresponding to the future data. Since the conventional correlation receiver is not *near-far resistant* this approximation brings in an element of near-far dependence, especially for small processing windows. The suitability of including the conventional receiver as a decision step (especially in MA limited conditions) is brought in to question. Instead we adopt the following approach based on the assumption that error control coding will be used in the system. We exploit the following property we have observed for a class of convolutional codes, for the purposes of this prediction.

For a code of constraint length $m + 1$ and code rate $R = \frac{1}{2}$, observing m consecutive pairs of outputs (2m bits), uniquely defines a path connecting $m + 1$ states, and hence uniquely defines the current $(m + 1)^{th}$ state on this path. If the next bit of output is also known, this path uniquely determines the second output bit.

By interleaving the data using a convolutional interleaver, and sliding the processing window by two vectors at each iteration, this prediction can be made to a high degree of accuracy.

Finally we use LU decomposition to exploit the block tridiagonal structure of the system matrix and reduce the solution to the following set of recurrences :

$$\begin{aligned}
Z_1 &= R(0)^{-1}R(-1), \\
Z_k &= (R(0)-R(1)Z_{k-1})^{-1}R(-1) \\
&\quad k=2,3\ldots\acute{N} \\
W_1 &= R(0)^{-1}\underline{y}(M+1), \\
W_k &= (R(0)-R(1)Z_{k-1})^{-1}\big(\underline{y}(M+k)-R(1)W_{k-1}\big) \\
&\quad k=2,3\ldots\acute{N} \\
\underline{WB}(M+\acute{N}) &= W_{\acute{N}}, \\
\underline{WB}(M+k) &= W_k - Z_k\underline{WB}(M+k+1) \\
&\quad k=\acute{N}-1,\acute{N}-2,\ldots,1 \qquad (11)
\end{aligned}$$

Note that the matrix inversions need only be performed once unless there is a change in the timing configuration. Figure 2 shows a schematic representation of our algorithm.

4 Simulation Results

A multi-user, asynchronous DS-CDMA model has been written using Gold codes of length 63. A coded data rate of 20 kBps (BPSK) is used, accommodating the half rate convolutional code mentioned above. In this paper we present results showing the capacity gain offered by our decorrelating receiver (DR) over the conventional linear correlation receiver (CR), in AWGN and fast fading channels at 20 kBps. The latter assumes a single Rayleigh fading link at 900 Mhz with no diversity gain available. Individual users undergo independent fading at a rate dependent on the vehicle speed. Our simulation uses differential detection as it is unlikely that the phase of each user could be tracked accurately enough to justify an assumption of coherent reception. Figure 3 establishes the near-far resistance of the DR in AWGN. It is clear that the DR has no irreducible error rate irrespective of the number of, or relative energies of interfering (INT) users. Also it offers large capacity gains over the CR especially in multiple access limited conditions, which are those encountered in practice. Figure 4 is a performance comparison in a fast fading environment. The performance of the DR is seen to be largely invariant with the number of users, and exhibits a large capacity improvement, especially as the channel gets more multiple access limited. Figure 5 shows the irreducible error rates for different vehicle speeds. It could be expected that perfect decorrelation is not achieved due to the rapid variations in the received energies and the corresponding impact on the prediction strategies used in the algorithm. This is seen from the nominal vari-

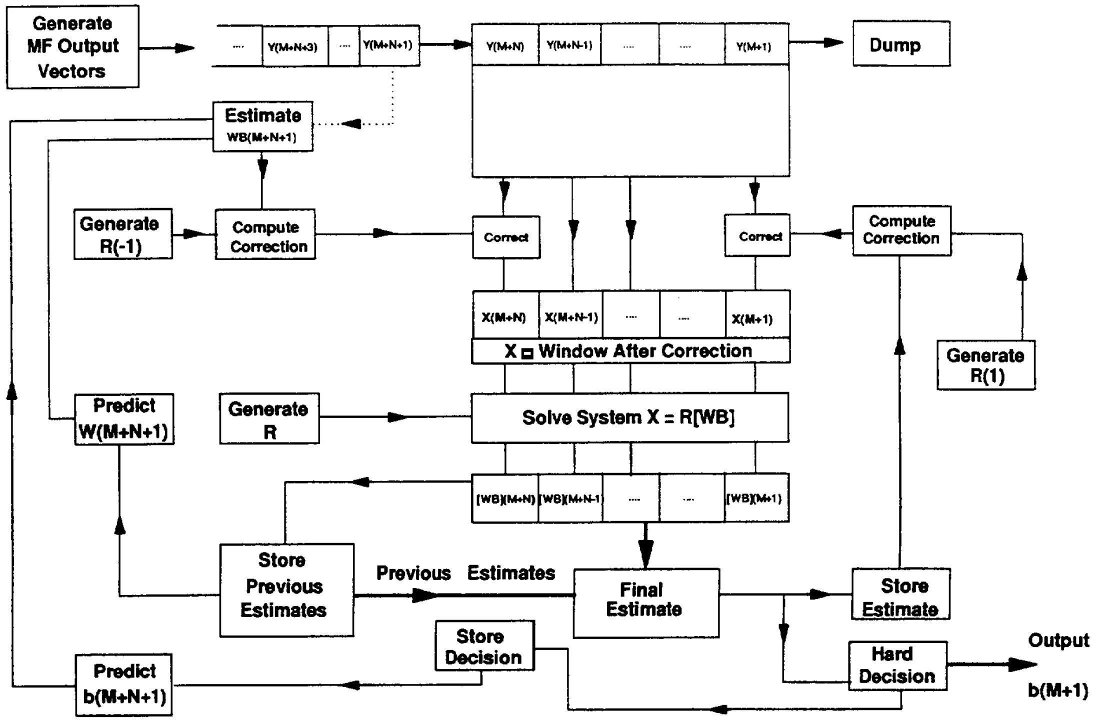

Figure 2: Sliding Window Decorrelating Algorithm

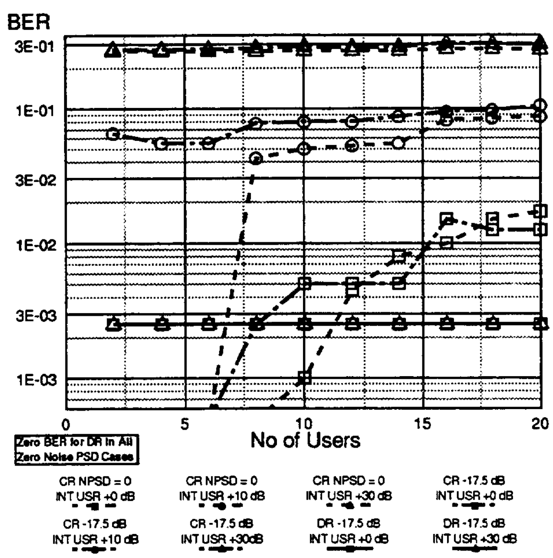

Figure 3: Performance in AWGN with Strong Interfering User

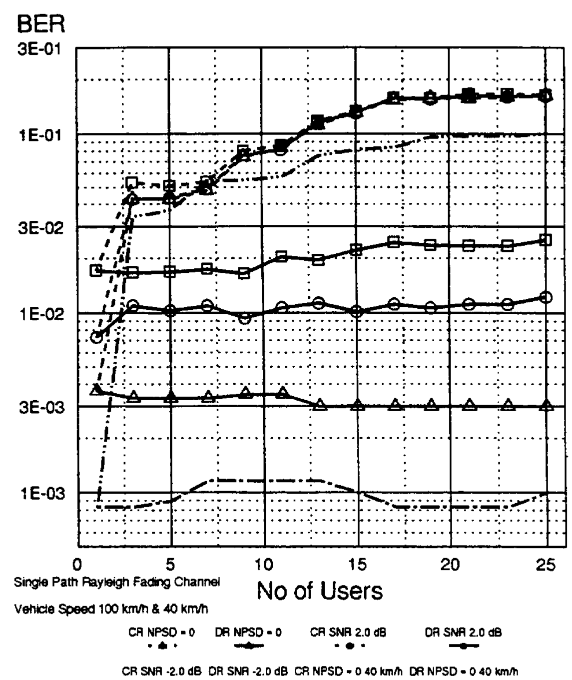

Figure 4: Multiple Users in Fading Channel

ations of the irreducible error curves, away from the single user irreducible error of the fading link.

Figure 6 shows the degradation of the DR performance as the channel gets noise limited as opposed to multiple access limited. The algorithm is seen to mis-interpret thermal noise as multi user interference. We hence establish that the algorithm should be targeted at operating conditions where multi user interference is far in excess of the receiver thermal noise. This operating condition is that which is found in practice for a multi user system. For purposes of clarity we have omitted the BER of the DR after decoding the convolutional code. It was verified that the BER of the DR was reduced to 0.0% - 0.3% whereas the BER of the CR was in excess of the error correcting capability of the half rate code for more than 5 users.

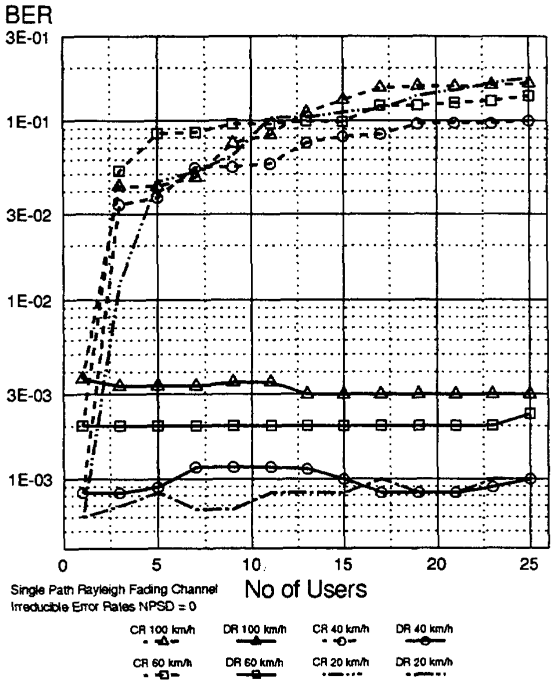

Figure 5: Irreducible Error in Fading Channel

5 Conclusions

We have shown by simulation that our algorithm is near-far resistant in AWGN and fast fading channels. In agreement with Turin [1], the CR is shown to be unusable in the absence of sufficiently rapid power control. The DR is hence shown to be a viable alternative to a complex closed loop power control strategy, the feasibility of which is questionable for practical vehicle speeds. We hence conclude that the Sliding Window Algorithm has potential application to future generation point-to-point and mobile radio DS-CDMA systems.

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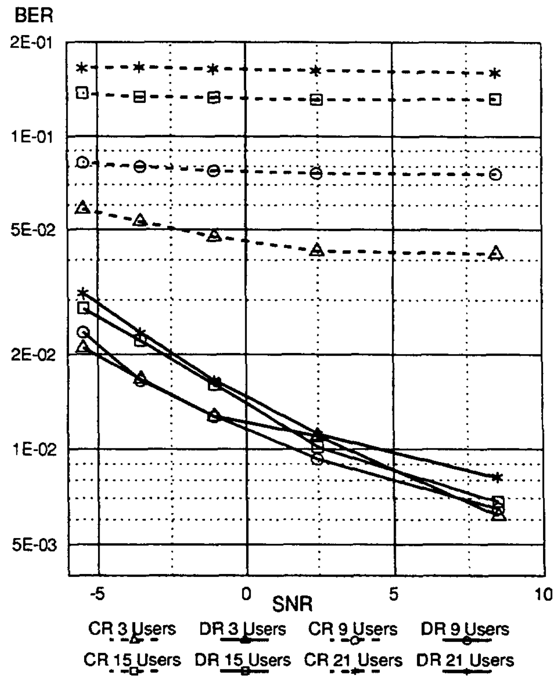

Figure 6: Noise Performance in Fading Channel

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